LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – APRIL 2010

ST 1815 / 1810 - ADVANCED DISTRIBUTION THEORY

Date & Time: 29/04/2010 / 1:00 - 4:00   Dept. No.   Max. : 100 Marks

SECTION – A   Answer all the questions   (10 x 2 = 20)

01. Find the mean of truncated geometric distribution, left truncated at zero.
02. Define a power series distribution and give an example.
03. Show that the ratio of two independent Lognormal variables is Lognormal.
04. State the pdf of Inverse Gaussian distribution.
05. Define a bivariate binomial distribution.
06. Let (X_1, X_2) have a bivariate Poisson distribution. Show that the correlation coefficient cannot be negative.
07. Show that the exponential distribution has a constant failure rate.
08. Let (X_1, X_2) have a bivariate normal distribution. Show that $X_1 - 2X_2$ has a normal distribution.
09. Let X_1, X_2 be independent standard normal variables. Examine whether X_1X_2 is distributed as chi-square.
10. Let X be P(θ), θ = 1,2,3. If θ is discrete uniform, find the mean of the compound distribution.

SECTION – B   Answer any five questions   (5 x 8 = 40)

11. State and establish a characterization of Bernoulli distribution based on raw moments.
12. State and establish the additive property satisfied by bivariate Poisson distribution.
13. Find the conditional distributions associated with Bivariate binomial distribution.
14. Derive the momemt generating function of Inverse Gaussian distribution.
15. For a Lognormal distribution, state and establish a relation between the median and the mode.
16. Let (X_1, X_2) have a Bivariate exponential distribution of Marshall-Olkin. Obtain a necessary and sufficient condition for the independence of X_1 and X_2.
17. Find the mean and the variance of non-central F - distribution.
18. Given a random sample from a standard normal , show that a quadratic form in the sample observatios is chi-square if the matrix of the quadratic form is idempotent.

SECTION – C   Answer any two questions   (2 x 20 = 40)

19 a) State and establish a characterization of geometric distribution based on lack of memory property.

b) Derive the moment generating function for a power-series distribution. Hence obtain the moment generating functions of Binomial and Poisson distributions.

20 a) Let X_1 and X_2 be two independent and identically distributed random variables with a finite variance. If $X_1 + X_2$ and $X_1 - X_2$ are independent, show that X_1 is normal.

b) Show that the conditional distributions associated with bivariate normal distribution are normal.

21a) State and establish a characterization of Marshall-Olkin bivariate exponential distribution based on bivariate lack of memory property.

b) Let X_1 and X_2 be independent Inverse Gaussian random variables. Construct a function of X_1 and X_2 which has a chi-square distribution.

22 a) Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed normal variables. Find the distribution of the sample variance and show that the sample mean is independent of the sample variance, using the theory of quadratic forms.

b) State and prove Cochran's theorem on quadratic forms in normal variables.

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